

① Given system of linear equations can be expressed as, $AX = 0$

where,

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \quad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, $AX = 0$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_2 \rightarrow \frac{1}{4}(R_2)$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

$$\text{No. of unknowns} = 4$$

$$\rho(A) < \text{no. of unknowns} = 4$$

$\therefore$ Given system of linear equations will have infinite solutions.

$$\text{Parameters} \rightarrow \text{Variables (Unknowns)} - \text{Rank} \\ = 4 - 2$$

$$= 2$$

i.e. infinitely many solutions are dependent on 2 free unknowns.

we know,

$$2x + y + w = 0 \quad \text{--- (1)}$$

$$z + w = 0 \quad \text{--- (2)}$$

from (2)

$$z = -w$$

let

$$z = k_1$$

so

$$w = -k_1$$

let

$$y = k_2$$

in eq (1)

$$2x + k_2 + k_1 = 0$$

$$2x = -k_1 - k_2$$

$$\boxed{x = -\frac{1}{2}(k_1 + k_2)}$$

$$\text{So, } x = -\frac{1}{2}(k_1 + k_2)$$

$$y = k_2$$

$$z = -k_1$$

$$w = k_1 //$$

② Given system of linear equations can be expressed as

$$AX = B \text{ where,}$$

$$A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 11 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

now,

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

Augmented Matrix $[A:B]$

$$= \begin{bmatrix} 5 & 3 & 7 & : & 4 \\ 3 & 26 & 2 & : & 9 \\ 7 & 2 & 11 & : & 5 \end{bmatrix}$$

$$R_2 \rightarrow 5R_2 - 3R_1$$

$$R_3 \rightarrow 5R_3 - 7R_1$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & : & 4 \\ 0 & 121 & -11 & : & 33 \\ 0 & -11 & 6 & : & -3 \end{bmatrix}$$

$$R_3 \rightarrow 11R_3 + R_2$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & : & 4 \\ 0 & 121 & -11 & : & 33 \\ 0 & 0 & 55 & : & 0 \end{bmatrix}$$

$$\rho(A) = \rho(A:B) = \text{no. of unknowns} = 3$$

So, given system of linear equations is consistent and has unique solution.

$$5x + 3y + 7z = 4 \quad (1)$$

$$121y - 11z = 33 \quad (2)$$

$$55z = 0 \quad (3)$$

$$\boxed{z=0} \text{ // from (3)}$$

in eq (2)

$$11(11y - z) = 33$$

$$11y - z = 3$$

$$(z=0)$$

$$11y = 3$$

$$\boxed{y = \frac{3}{11}} =$$

in eq (1)

$$5x + \frac{3}{11} + 0 = 4$$

$$5x = \frac{35}{11}$$

$$\boxed{x = \frac{7}{11}} =$$

$$\frac{\sinh(x)}{\cosh^2(x)}$$

③ Given system of linear equations can be expressed as, $AX=B$

where, $A = \begin{bmatrix} 2 & -3 & 6 & -5 \\ 0 & 1 & -4 & 1 \\ 4 & -5 & 8 & -9 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$

$$B = \begin{bmatrix} 3 \\ 1 \\ k \end{bmatrix}$$

So,

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 6 & -5 \\ 0 & 1 & -4 & 1 \\ 4 & -5 & 8 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ k \end{bmatrix}$$

Augmented Matrix $[A:B]$

$$= \begin{bmatrix} 2 & -3 & 6 & -5 & : & 3 \\ 0 & 1 & -4 & 1 & : & 1 \\ 4 & -5 & 8 & -9 & : & k \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\sim \begin{bmatrix} 2 & -3 & 6 & -5 & : & 3 \\ 0 & 1 & -4 & 1 & : & 1 \\ 0 & 1 & -4 & 1 & : & k-6 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\sim \begin{bmatrix} 2 & -3 & 6 & -5 & : & 3 \\ 0 & 1 & -4 & 1 & : & 1 \\ 0 & 0 & 0 & 0 & : & k-7 \end{bmatrix}$$

(i) for no solution,

$$\rho(A) \neq \rho(A:B)$$

we know,

$$\rho(A) = 2$$

$$\text{for } \rho(A:B) \neq 2,$$

$$k-7 \neq 0$$

$$\boxed{k \neq 7}$$

(ii) for infinite solutions,

$$\rho(A) = \rho(A:B) < \text{no. of unknowns} = 4$$

we know,

$$\rho(A) = 2$$

$$\text{for } \rho(A:B) = 2,$$

$$k-7=0$$

$$\boxed{k=7}$$

(iii) for unique solutions,

$$\rho(A) = \rho(A:B) = \text{no. of unknowns} = 4$$

we know,

$$\rho(A) = 2$$

$$\text{but no. of unknowns} = 4$$

So, for any value of k , the given system of linear equations cannot have a unique solution.

sum
ms.

⑤ Given system of linear equations can be expressed as $AX=B$,

where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \alpha \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 10 \\ \beta \end{bmatrix}$$

Now,

$$AX=B$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \beta \end{bmatrix}$$

Augmented Matrix $(A:B)$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \alpha & : & \beta \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \alpha-1 & : & \beta-6 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \alpha-3 & : & \beta-10 \end{bmatrix}$$

(i) for no solution,

$$\rho(A) \neq \rho(A:B)$$

So, ~~we~~

$$\alpha - 3 = 0$$

$$\boxed{\alpha = 3}$$

and

$$\beta - 10 \neq 0$$

$$\boxed{\beta \neq 10}$$

(ii) for unique solutions,

$$\rho(A) = \rho(A:B) = \text{no. of unknowns} = 3$$

so,

$$\alpha - 3 \neq 0$$

$$\boxed{\alpha \neq 3}$$

β can be any value

(iii) for infinite solutions / more than one solution

$$\rho(A) = \rho(A:B) < \text{no. of unknowns} = 3$$

$$\alpha - 3 = 0$$

$$\boxed{\alpha = 3}$$

$$\beta - 10 = 0$$

$$\boxed{\beta = 10}$$

(6) given system of linear equations can be expressed as

$$AX = B, \text{ where,}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$$

Now
 $AX=B,$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$$

Augmented Matrix $(A:B)$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 5 \\ 1 & -1 & 1 & 0 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 0 & -2 & 0 & -4 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 2 & 2 \end{array} \right]$$

$$\rho(A) = \rho(A:B) = \text{no. of unknowns} = 3$$

So, given system of linear equations will have
unique solution.

$$x + y + z = 4 \quad (1)$$

$$-y - z = -3 \quad (2)$$

$$2z = 2 \quad (3)$$

$$\boxed{z=1} \text{ from (3)}$$

in (2),

$$-y - 1 = -3$$

$$-y = -3 + 1$$

$$-y = -2$$

$$\boxed{y=2}$$

in (1),

$$x + 1 + 2 = 4$$

$$\boxed{x=1}$$

⑦ Given system of linear equations can be expressed as $AX=B$,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \\ 3 & 9 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 4 \end{bmatrix}$$

$$AX=B$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \\ 3 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 4 \end{bmatrix}$$

Augmented Matrix $(A:B)$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 2 & : & 5 \\ 3 & -5 & 5 & : & 2 \\ 3 & 9 & -1 & : & 4 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$R_4 \rightarrow R_4 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & -1 & 0 & : & -1 \\ 0 & -11 & 2 & : & -7 \\ 0 & 3 & -4 & : & -5 \end{bmatrix}$$

$$R_2 \rightarrow (-1)R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & -11 & 2 & : & -7 \\ 0 & 3 & -4 & : & -5 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 11R_2$$

$$R_4 \rightarrow R_4 - 3R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 2 & : & 4 \\ 0 & 0 & -4 & : & -8 \end{bmatrix}$$

$$R_3 \rightarrow \left(\frac{1}{2}\right)R_3$$

$$R_4 \rightarrow \left(\frac{1}{4}\right)R_4$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 2 \\ 0 & 0 & 1 & : & 2 \end{bmatrix} \quad 2$$

$$R_4 \rightarrow R_4 - R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 2 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$x + 2y + z = 3 \quad (1)$$

~~2x + 2y~~ $y = 1 \quad (2)$

$$z = 2 \quad (3)$$

Put (2), (3) in (1)

$$x + 2 + 2 = 3$$

$$\boxed{x = -1} =$$



⑧ Given system of linear equations can be expressed as $AX = B$,

$$A = \begin{bmatrix} 3 & -1 & k \\ 2 & 1 & 1 \\ 1 & 2 & -k \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 3 & -1 & k \\ 2 & 1 & 1 \\ 1 & 2 & -k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Augmented matrix $(A:B)$

$$= \begin{bmatrix} 3 & -1 & k & : & 1 \\ 2 & 1 & 1 & : & 2 \\ 1 & 2 & -k & : & -1 \end{bmatrix}$$

$$R_3 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -k & : & -1 \\ 2 & 1 & 1 & : & 2 \\ 3 & -1 & k & : & 1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -k & : & -1 \\ 0 & -3 & 1+2k & : & 4 \\ 0 & -7 & 4k & : & 4 \end{bmatrix}$$

$$R_3 \rightarrow (-3)R_3 + 7R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -k & : & -1 \\ 0 & -3 & 1+2k & : & 4 \\ 0 & 0 & 2k+7 & : & 16 \end{bmatrix}$$

(i) For not having a unique solution,

$$\rho(A) \neq \rho(A:B)$$

$$\text{we know, } \rho(A:B) = 3$$

$$\text{for } \rho(A) \neq 3$$

$$2k+7=0$$

$$2k = -7$$

$$\boxed{k = -\frac{7}{2}}$$

(ii) with $k = -\frac{7}{2}$, the system of linear equations has no solution,

$$\text{as, } 0x + 0y + 2k+7 = 16$$

$$\text{if we take } k = -\frac{7}{2}$$

$$0 \neq 16$$



no solution,

⑨ Given system of linear equations can be expressed as

$$AX = 0$$

where, $A = \begin{bmatrix} 1 & 1 & -2 & 3 \\ 1 & -2 & 1 & -1 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$ $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$AX = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -2 & 3 \\ 1 & -2 & 1 & -1 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -2 & 3 \\ 1 & -2 & 1 & -1 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - 4R_1$$

$$R_4 \rightarrow R_4 - 5R_1$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & -4 \\ 0 & -3 & 3 & -4 \\ 0 & -12 & 12 & -16 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & -12 & 12 & -16 \end{bmatrix}$$

$$R_u \rightarrow R_u - 4R_2$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

$$\rho(A) < \text{no. of unknowns (3)}$$

$\therefore$ equations have infinite solutions

$$x + y - 2z + 3w = 0 \quad (1)$$

$$-3y + 3z - 4w = 0 \quad (2)$$

from (2) $\boxed{y = z - \frac{4}{3}w} =$

let $z = t_1$, $w = t_2$

$$x + t_1 - \frac{4}{3}t_2 - 2t_1 + 3t_2 = 0$$

$$\boxed{x = t_1 - \frac{5}{3}t_2} =$$

So, $\boxed{y = t_1 - \frac{4}{3}t_2} =$

$$\boxed{z = t_1} =$$

$$\boxed{w = t_2} =$$

(10) Given system of linear equations can be expressed as, $AX=0$

$$\text{where, } A = \begin{bmatrix} 3 & 4 & -1 & -6 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 1 & 3 & 13 & 3 \end{bmatrix} \text{ \& } X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \text{ \& } 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX=0$$

$$\Rightarrow \begin{bmatrix} 3 & 4 & -1 & -6 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 1 & 3 & 13 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 & -1 & -6 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 1 & 3 & 13 & 3 \end{bmatrix}$$

$$R_4 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 3 & 13 & 3 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -9 \\ 3 & 4 & -1 & -6 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 2R_1, R_4 \rightarrow R_4 - 4R_1$$

$$\sim \begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & -3 & -24 & -9 \\ 0 & -5 & -40 & -15 \\ 0 & -5 & -40 & -15 \end{bmatrix}$$

$$R_2 \rightarrow \begin{pmatrix} 1 \\ -3 \end{pmatrix} R_2, R_3 \rightarrow \begin{pmatrix} -1 \\ 5 \end{pmatrix} R_3, R_4 \rightarrow \begin{pmatrix} 1 \\ 5 \end{pmatrix} R_4$$

$$\sim \begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 1 & 8 & 3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_4 \rightarrow R_4 - R_3$$

$$\sim \begin{bmatrix} 1 & 3 & 13 & 3 \\ 0 & 1 & 8 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

$$\rho(A) < \text{no. of unknowns (4)}$$

so there are infinite solutions

$$1x + 3y + 13z + 3w = 0 \quad -(1)$$

$$1y + 8z + 3w = 0 \quad -(2)$$

$$\boxed{y = -(8z + 3w)}$$

let

$$z = k_1$$

$$w = k_2$$

from (1)

$$\boxed{x = 11k_1 + 6k_2}$$

$$\boxed{y = -8k_1 - 3k_2}$$

$$\boxed{z = k_1} \quad \boxed{w = k_2} =$$